GENERATION OF RAINFALL INTENSITY-DURATION-FREQUENCY RELATIONSHIP FOR CENTRAL REGION IN BANGLADESH

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ABSTRACT

The objective of this research was to derive Rainfall IDF relationship for Central region of Bangladesh. Two common frequency analysis techniques Gumbel and Log Pearson Type III (LPTIII) distribution were used to develop the IDF relationship from rainfall data of this region. Yearly maximum rainfall data for last 41 years (1974-2014) from Bangladesh Meteorological Department (BMD) was used in this study. Indian Meteorological Department (IMD) empirical reduction formula was used to estimate the short duration rainfall intensity from yearly maximum rainfall data. The results obtained using Gumbel method are slightly higher than LPT III distribution method. Rainfall intensities obtained from these two methods showed good agreement with results from previous studies on some parts of the study area. The chi-square goodness of fit test was used to determine the best fit probability distribution. The parameters of the IDF equations and coefficient of correlation for different return periods (2, 5, 10, 25, 50 and 100 years) are calculated by using nonlinear multiple regression method. The results obtained presented that in all the cases the correlation coefficient is very high representing the goodness of fit of the formulae to estimate IDF curves in the region of interest. It was found that intensity of rainfall decreases with increase in rainfall duration. Further, a rainfall of any given duration will have a larger intensity if its return period is large. In other words, for a rainfall of given duration, rainfalls of higher intensity in that duration are rarer than rainfalls of smaller intensity.

Keywords: *Rainfall intensity, rainfall frequency, Bangladesh Meteorological Department (BMD), Gumbel's Extreme Value distribution, LPTIII distribution*

1. INTRODUCTION

Rainfall intensity-duration-frequency (IDF) curves are graphical exemplifications of the amount of water that falls within a given period of time in catchment areas (Dupont and Allen, 2000). IDF curves are used to aid the engineers while designing urban drainage works. The establishment of such relationships was done as early as 1932 (Chow, 1988 and Dupont and Allen, 2006). Since then, many sets of relationships have been constructed for several parts of the globe. However, such relationships have not been accurately constructed in many developing countries (Koutsoyiannis et al.,1998). Koutsoyiannis et al., 1998; Koutsoyiannis, 2003 cited that the IDF relationship is a mathematical relationship between the rainfall intensity I, the duration T_d, and the return period T_r. Indeed the IDF-curves allow for the estimation of the return period of an observed rainfall event or conversely of the rainfall amount corresponding to a given return period for different aggregation times.

In Kentucky, for example IDF curves are used in conjunction with runoff estimation formulae; e.g. the Rational Method, to predict the peak runoff amounts from a particular watershed. The information from the curves is then used in hydraulic design to size culverts and pipes (Dupont and Allen, 2000). Further studies by (Ilona and Frances, 2002) performed rainfall analysis and regionalization of IDF curves for different regions.

In Bangladesh water logging and flood is a common problem during Monsoon period because of inadequate drainage system. In order to solve this problem new drainage design is needed where rainfall data of different duration is needed. But due to instrumental limitation these data were not available. In the present study, annual maximum rainfall series is considered for Rainfall Frequency Analysis (RFA). Rainfall in a region can be characterized if the intensity, duration and frequency of the diverse storms occurring at that place are known. The frequency-data for rainfalls of various durations, so obtained, can be represented by IDF curves, which give a plot of rainfall intensity versus rainfall duration and recurrence interval. In recent studies, various authors attempted to relate IDF relationship to the synoptic meteorological conditions in the area of hydrometric stations (Dupont and Allen, 2006).

Matin et al.,1984, in their study developed the IDF curve for North-East region of Bangladesh and also observed that the rainfall data in this region follow Gumbel's Extreme Value Distribution. Chowdhury et al.,2007, developed the short duration rainfall IDF curve for Sylhet with return period of 2, 5, 10, 20, 50, and 100 years. Kim et al. (2008) improved the accuracy of IDF curves by using long and short duration separation technique. They derived IDF curves by using cumulative distribution function (CDF) for the site under consideration using multi-objective genetic algorithm. Khaled et al. (2011) applied L-moments and generalized least squares regression methods for estimation of design rainfall depths and development of IDF relationships. Rashid et al. (2012) applied Pearson Type-III distribution for modelling of short duration rainfall and development of IDF relationships for sylhet city in Bangladesh. In probability theory, extreme value distributions namely Gumbel, Frechet and Weibull are generally considered for frequency analysis of meteorological variables. On the other hand, Atomic Energy Regulatory Board (AERB) guidelines described that the Order Statistics Approach (OSA) is the most appropriate method for determination of parameters of Gumbel and Frechet distributions. In this present study Gumbel's Extreme Value Distribution method is used to develop IDF curves and equations. In this context, an attempt has been made to estimate the rainfall for different return periods for different durations of 'n' such as 10-min, 20-min, 30-min, 60-min, 120-min, 180-min, 360-min, 720-min, 1440-min adopting Gumbel distributions for development of IDF relationships for seven divisions of Bangladesh. Model performance indicators (MPIs) such as correlation coefficient (R) was used to analyze the performance of the developed IDF relationships by Gumbel distributions for estimation of rainfall intensity for the stations under study.

2. DATA COLLECTION AND METHODOLOGY

For this study 24 hr daily rainfall data from year 1974 to 2014 was collected from Bangladesh Meteorological Department (BMD) for Central regions. From the daily data maximum yearly rainfall data was used in the analysis. In Central regions there are six BMD stations (Dhaka, Comilla, M.Court, Mymensingh, Tangail, Chandpur) which are taken into consideration to develop IDF curve for central region of Bangladesh. For accurate hydrologic analyses, reliable rainfall intensity estimates are necessary. The IDF relationship comprises the estimates of rainfall intensities of different durations and recurrence intervals. There are commonly used theoretical distribution functions that were applied in different regions all over the world; (e.g. Generalized Extreme Value Distribution (GEV), Gumbel, Pearson type III distributions) (Dupont and Allen, 2000). Two common frequency analysis techniques were used to develop the relationship between rainfall intensity, storm duration, and return periods from rainfall data for the regions under study. These techniques are: Gumbel distribution and LPT III distribution.

2.1 Estimation of Short Duration Rainfall

Indian Meteorological Department (IMD) use an empirical reduction formula (equation (1)) for estimation of various duration like 1-hr, 2-hr, 3-hr, 5-hr, 8-hr rainfall values from annual maximum values. Chowdhury et al (2007), used Indian Meteorological Department (IMD) empirical reduction formula to estimate the short duration rainfall from daily rainfall data in Sylhet city and found that this formula give the best estimation of short duration rainfall. In this study this empirical formula (equation (1)) was used to estimate short duration rainfall of six stations of Central region of Bangladesh.

$$P_t = P_{24} \sqrt[3]{\frac{t}{24}} \tag{1}$$

Where, P_t is the required rainfall depth in mm at t-hr duration, P_{24} is the daily rainfall in mm and t is the duration of rainfall for which the rainfall depth is required in hr.

2.2 Gumbel Theory of Distribution

For Gumbel distribution methodology was selected to perform the flood probability analysis. The Gumbel theory of distribution is the most widely used distribution for IDF analysis owing to its suitability for modelling maxima. It is relatively simple and uses only extreme events (maximum values or peak rainfalls). The Gumbel method calculates the 2, 5, 10, 25, 50 and 100 year return intervals for each duration period and requires several calculations. Frequency precipitation P_T (in mm) for each duration with a specified return period T (in year) is given by the following equation:

$$P_T = P_{ave} + KS \tag{2}$$

Where K is Gumbel frequency factor given by:

$$K = -\frac{\sqrt{6}}{\pi}[0.5772 + \ln[\ln[\frac{T}{T-1}]]] \tag{3}$$

Where P_{ave} is the average of the maximum precipitation corresponding to a specific duration.
In utilizing Gumbel's distribution, the arithmetic average in equation (2) is used:

$$P_{ave} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Pi} \tag{4}$$

Where Pi is the individual extreme value of rainfall and n is the number of events or years of record. The standard deviation is calculated by equation (5) computed using the following relation:

$$S = [\frac{1}{n-1}\sum_{i=1}^{n}(\mathrm{Pi} - \mathrm{Pave})^2]^{1/2} \tag{5}$$

Where S is the standard deviation of P data. The frequency factor (K), which is a function of the return period and sample size, when multiplied by the standard deviation gives the departure of a desired return period rainfall from the average. Then the rainfall intensity, I_T (in mm/h) for return period T is obtained from:

$$I_T = \frac{P_T}{Td} \tag{6}$$

Where T_d is duration in hours.
The frequency of the rainfall is usually defined by reference to the annual maximum series, which consists of the largest values observed in each year. An alternative data format for rainfall frequency studies is that based on the peak-over threshold concept, which consists of all precipitation amounts above certain thresholds selected for different durations. Due to its simpler structure, the annual-maximum-series method is more popular in practice (Bogra, Vezzani and Fontana, 2005)

2.3 Log Pearson Type III

The LPT III probability model is used to calculate the rainfall intensity at different rainfall durations and return periods to form the historical IDF curves for each station. LPT III distribution involves logarithms of the measured values. The mean and the standard deviation are determined using the logarithmically transformed data. In the same manner as with Gumbel method, the frequency precipitation is obtained using LPT III method. The simplified expression for this latter distribution is given as follows:

$$P^* = \log(\mathrm{Pi}) \tag{7}$$

$$P_T^* = Pave^* + K_T S^* \tag{8}$$

$$Pave^* = \frac{1}{n}\sum_{i=1}^{n} P^* \tag{9}$$

$$S^* = \left[\frac{1}{n-1}\sum_{i=1}^{n}(P^* - Pave^*)^2\right]^{1/2} \tag{10}$$

Where P_T^*, P_{ave}^*, S^* are logarithmic extreme value of rainfall, average of maximum precipitation corresponding to a specific duration, standard deviation of P* data respectively. K_T is the Pearson frequency factor which depends on return period (T) and skewness coefficient (Cs). The skewness coefficient, Cs, is required to compute the frequency factor for this distribution. The skewness coefficient is computed by equation (11).

$$C_S = \frac{n\sum_{i=1}^{n}(Pi^* - Pave^*)^3}{(n-1)(n-2)(S^*)^3} \tag{11}$$

K_T values can be obtained from tables in many hydrology references. By knowing the skewness coefficient and the recurrence interval, the frequency factor, K_T for the LPT III distribution can be extracted. The antilog of the solution in equation (7) will provide the estimated extreme value for the given return period.

2.4 Derivation of IDF Equation

The IDF formulae are the empirical equations representing a relationship between maximum rainfall intensity as a dependent variable and other parameters of interest; for example the rainfall duration and frequency as independent variables. There are several commonly used functions relating those variables previously mentioned found in the literature of hydrology applications (Chow, 1988; Burke and Burke, 2008).
To derive an equation for calculating the rainfall intensity (I) for the regions of interest, there are some required steps for establishing an equation to suit the calculation of rainfall intensity for a certain recurrence interval and specific rainfall period which depends mainly on the results obtained from the IDF curves. Two approaches were tried to estimate the equation parameters.

A. By applying the logarithmic conversion, where it is possible to convert the equation into a linear equation, thus to calculate all the parameters related to the equation. The following steps are followed:

I. Convert the original equation in the form of power-law relation (Chow, 1988; Koutsoyiannis et al., 1998) as follows:

$$I = \frac{CT_r^{\,m}}{T_d^{\,e}} \tag{12}$$

By applying the logarithmic function to get

$$log\ I = log\ K - e\ log\ T_d \tag{13}$$

Where

$$K = CTr^m \tag{14}$$

And e represents the slope of the straight line.

II. Calculate the natural logarithm for (K) value found from Gumbel method or from LPTIII method as well as the natural logarithmic for rainfall period T_d.

III. Plot the values of (log I) on the y-axis and the value of (log T_d) on the x axis for all the recurrence intervals for the two methods.

IV. From the graphs (or mathematically) find the value of (e) for all recurrence intervals. Then it was found out the average value of e value, e_{ave}, by using the following equation

$$e_{ave} = \frac{\sum e}{n} \tag{15}$$

Where n represents recurrence intervals (years) value noted as Tr.

V. From the graph, it was found (*logK*) values for each recurrence interval where (*logK*) represents the Y-intercept values as per Gumbel method or LPTIII method. Then convert equation (14) into a linear equation by applying the natural logarithm to become:

$$log\ K = logC + m\ logTr \tag{16}$$

VI. Plot the values of (*logK*) on the y-axis and the values of (*logTr*) on the x-axis to find out the values of parameters c and m as per Gumberl method or LPTIII where (m) represents the slope of the straight line and (C) represents the (anti log) for the y intercept.

B. Estimation of the equation parameters by using nonlinear regression analysis: Using the Solver function of the ubiquitious spreadsheet programme Microsoft Excel, which employs an iterative least squares fitting routine to produce the optimal goodness of fit between data and function. The R^2 value calculated is designed to give the user an estimate of goodness of fit of the function to the data.

2.5 Goodness of Fit Test

The aim of the test is to decide how good is a fit between the observed frequency of occurrence in a sample and the expected frequencies obtained from the hypothesized distributions. A goodness of fit test between observed and expected frequencies is based on the chi-square quantity, which is expressed as,

$$\chi^2 = \sum_{i=1}^{k} (\mathrm{O_i} - \mathrm{E_i})^2 / \mathrm{E_i} \tag{17}$$

Where χ^2 is a random variable whose sampling distribution is approximated very closely by the chi-square distribution. The symbols Oi and Ei represent the observed and expected frequencies, respectively, for the i-th class interval in the histogram. The symbol k represents the number of class intervals. If the observed frequencies are close to the corresponding expected frequencies, the χ^2 value will be small, indicating a good fit; otherwise, it is a poor fit. A good fit leads to the acceptance of null hypothesis, whereas a poor fit leads to its rejection. The critical region will, therefore, fall in the right tail of the chi-square distribution. For a level of significance equal to a, the critical value is found from readily available chi-square tables and χ^2> constitutes the critical region.

3. RESULTS AND DISCUSSION

The purpose of this study was to develop IDF curves and derive an empirical formula to estimate the rainfall intensity at Central region in Bangladesh. The IDF curves are used as an aid when designing drainage structures for any engineering project. The curves allow the engineer to design safe and economical flood control measures. Rainfall estimates in mm and their intensities in mm/hr for various return periods and different durations were analysed using the two techniques: (Gumbel and LPT III). According to the IDF curves, rainfall estimates are increasing with increase in the return period and the rainfall intensities decrease with rainfall duration in all return periods. Rainfall intensities rise in parallel with the rainfall return periods. The results obtained from the two methods have good consistency.

Table 1: The parameters values used in deriving formula

Region	Parameter	Gumbel	Log Pearson III
Central Region	c	791	726
	m	0.26	0.28
	e	0.66	0.66

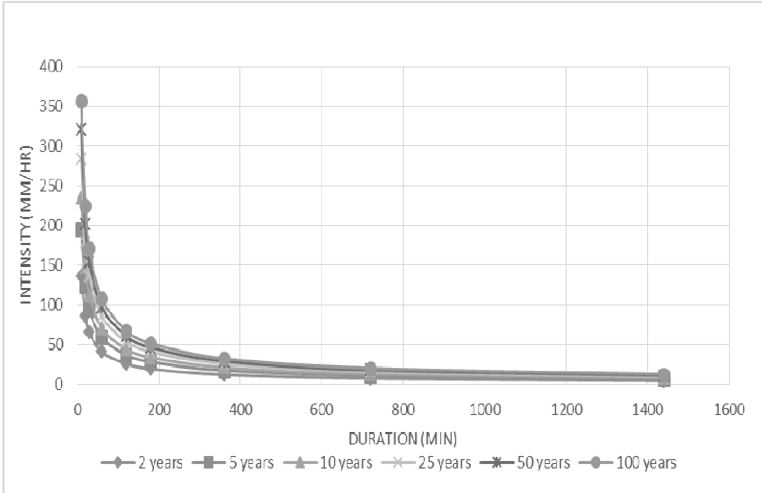

Figure 1: IDF curve by average of Gumbel and LPTIII methods at Central Region

Figure 1 shows result of the IDF curves obtained by average of Gumbel and LPT III methods for Central region. It was shown that there were small differences between the results obtained from the two methods, where Gumbel method gives slightly higher results than the results obtained by Log Pearson III. This is shown also from parameters of the derived equation for calculating the rainfall intensity using the two methods. Parameters of the selected IDF formula were adjusted by the method of minimum squares, where the goodness of fit is judged by the correlation coefficient. The results obtained showed that in all the cases the correlation coefficient is very high, and ranges between 0.998 and 0.994, except few cases where it ranges between 0.98 and 0.978 when using LPT III at 50 and 100 years. This indicates the goodness of fit of the formulae to estimate IDF curves in the region of interest. For each region the results are given as the mean value of the points results. Table 1 shows the parameters values obtained by analyzing the IDF data using the two methods and those are used in deriving formulae for the two regions. Also, goodness-of-fit tests were used to choose the best statistical distribution among those techniques. Results of the chi- square goodness of fit test on annual series of rainfall are shown in Table 2. As it is seen most of the data fit the distributions at the level of significance of α =0.05, which yields χ_{cal}<3.84. Only the data for 10 min and 20 min do not give good fit using Gumbel method distribution. Also the data for 10 min using LPTIII method do not give good fit.

Table 2: Results of chi-square goodness of fit test on annual maximum rainfall

Region	**Distribution**	**Duration in minutes**								
		10	20	30	60	120	180	360	720	1440
Central	Gumbel	6.69	3.99	2.94	1.75	1.0	0.76	0.45	0.27	0.16
	LPTIII	4.06	2.45	1.78	1.04	0.60	0.43	0.25	0.14	0.08

4. CONCLUSIONS

This research presents some insight into the way in which the rainfall is estimated in Bangladesh. Since Bangladesh has different climatic conditions from region to region, a relation for each region has to be obtained to estimate rainfall intensities for different durations and return periods ranging between 2 and 100 years. This study has been conducted for the formulation and construction of IDF curves using data from recording stations by using two distribution methods: Gumbel and LPT III distribution. Gumbel method gave some larger rainfall intensity estimates compared to LPT III distribution. In general, the results obtained using the two approaches are very close at most of the return periods and have the same trend, this agrees with the results obtained by Rashid et al., 2012. The results obtained from that work are consistent with the results from previous studies done in some parts of the study area. It is concluded that the difference observed between the results of this study and the results done before by Rashid et al. (2012) are accepted and in good agreement, and this can be attributed to the record lengths of the rainfall data used for this study and the studies before. The parameters of the design storm intensity for a given period of recurrence were estimated for each region. The results obtained showed a good match between the rainfall intensity computed by the methods used and the values estimated by the calibrated formulae. The results showed that in all the cases data fitted the formula with a correlation coefficient greater than 0.97. This indicates the goodness of fit of the formulae to estimate IDF curves in the region of interest for durations varying from 10 to 1440 min and return periods from 2 to 100 years. The chi-square test was used on one hand to examine the combinations or contingency of the observed and theoretical frequencies, and on the other hand, to decide about the type of distribution which the available data set follows. The results of the chi-square test of goodness of fit showed that in all the durations the null hypothesis that the extreme rainfall series have the Gumbel distribution is acceptable at the 5% level of significance.

Only few cases in which the fitting was not good obtained by using the LPT III distribution. Although the chi-square values are appreciably below the critical region using Gumbel distribution and few values are higher than the critical region using LPT III distribution, it is difficult to say that one distribution is superior to the other. Further studies are recommended whenever there will be more data to verify the results obtained or update the IDF curves.

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